

Semilattice orders on the homomorphic images of the Rédei semigroup

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The simple combinatorial principal ideal semigroups generated by two elements have been described by L. Megyesi and G. Pollák [2]. Their generators a, b fulfil the equations $a^2b = a$ and $ab^2 = b$. The semigroup defined by these two equations is called Rédei semigroup, because it appeared in [3] in another context. The "smallest" simple combinatorial principal ideal semigroup generated by two elements is the bicyclic semigroup. D. B. McAlister determined the compatible semilattice orders on the bicyclic semigroup in [1]. Our aim is to give semilattice orders on the homomorphic images of the Rédei semigroup. We prove that there are four total orders on these semigroups. We show that if \leq is a compatible semilattice order on the Rédei semigroup then \leq is necessarily a total order. Moreover we give a compatible semilattice order on each proper homomorphic image of the Rédei semigroup which is not a total order.

References

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- [3] L. Rédei, *Halbgruppen und Ringe mit Linkseinheiten ohne Linkseinselemente*, *Acta Math. Acad. Sci. Hungar.* 11 (1960), 217-222.